Applying ant colony optimization metaheuristic to solve forest transportation planning problems with side constraints

Marco A. Contreras, Woodam Chung, and Greg Jones

Abstract: Forest transportation planning problems (FTPP) have evolved from considering only the financial aspects of timber management to more holistic problems that also consider the environmental impacts of roads. These additional requirements have introduced side constraints, making FTPP larger and more complex. Mixed-integer programming (MIP) has been used to solve FTPP, but its application has been limited by the difficulty of solving large, real-world problems within a reasonable time. To overcome this limitation of MIP, we applied the ant colony optimization (ACO) metaheuristic to develop an ACO-based heuristic algorithm that efficiently solves large and complex forest transportation problems with side constraints. Three hypothetical FTPP were created to test the performance of the ACO algorithm. The environmental impact of forest roads represented by sediment yields was incorporated into the economic analysis of roads as a side constraint. Four different levels of sediment constraints were analyzed for each problem. The solutions from the ACO algorithm were compared with those obtained from a commercially available MIP solver. The ACO solutions were equal to or slightly worse than the MIP solution, but the ACO algorithm took only a fraction of the computation time that was required by the MIP solver.

Résumé : Les problèmes de planification du transport forestier (PPTF) ont évolué de la prise en considération uniquement des aspects financiers de la gestion forestière vers une approche plus globale qui tient compte aussi de l'effet des routes sur l'environnement. Ces exigences additionnelles ont amené des contraintes supplémentaires qui rendent les PPTF plus volumineux et plus complexes. La programmation linéaire mixte (PLM) a été utilisée pour résoudre les PPTF, mais ses applications ont été limitées par la difficulté à résoudre des problèmes du monde réel de grandes tailles, à l'intérieur d'un délai raisonnable. Pour surmonter cette faiblesse de la PLM, nous avons appliqué la métaheuristique de la colonie de fourmis pour développer un algorithme qui résout de façon efficace les PPTF complexes et de grandes tailles avec des contraintes complémentaires. Trois PPTF hypothétiques ont été créés pour tester la performance de l'algorithme par colonie de fourmis (ACF). L'impact environnemental des routes forestières, exprimé par la production de sédiments, a été incorporé dans l'analyse économique des routes comme une contrainte complémentaire. Quatre niveaux différents des contraintes de sédimentation ont été analysés pour chaque problème. Les solutions obtenues avec l'ACF ont été comparées à celles obtenues avec un résolveur PLM commercial. Les solutions de l'ACF étaient équivalentes ou légèrement pires que la solution de la PLM, mais elles ne nécessitaient qu'une fraction du temps de calcul requis par le résolveur PLM.

[Traduit par la Rédaction]

Introduction

Problems related to forest transportation planning have long been an important concern because timber transportation is one of the most expensive activities in forest operations (Greulich 2003). Traditionally, the goal of forest transportation planning problems (FTPP) has been to find the road network that minimizes both log hauling and road construction costs for timber management.

FTPP that contain both fixed (road construction) and variable (log hauling) costs of timber transportation are a special case of the fixed charge transportation problem (FCTP) (Balinski 1961), which is known as a NP-hard (nondeterministic polynomial-time hard) combinatorial optimization problem (Maniezzo et al. 1998; Kowalski 2005). These transportation problems have been solved optimally using exact algorithms such as stage ranking and branch-and-bound methods used in mixed-integer programming (MIP) (Adlakha and Kowalski 2003). However, the application of these methods has been restricted to small-scale problems because computation time dramatically increases with problem size (Kowalski 2005). For even a medium-scale problem with hundreds to thousands of edges, the computation time required by MIP might be so large that the problem becomes unsolvable for practical purposes (Adlakha and Kowalski 2003).

Several approximation algorithms, generally called heuristics, have been developed to solve larger problems in a reasonable time (Gottlieb and Paulmann 1998; Sun et al. 1998). Although heuristic algorithms may not always provide optimal solutions, they have been the focus of a large number of researchers because of their high efficiency and capability of

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M.A. Contreras and W. Chung.[1] Department of Forest Management, College of Forestry and Conservation, The University of Montana, Missoula, MT 59812, USA.
G. Jones. Social, Economics, and Decision Science Program, Rocky Mountain Research Station, USDA Forest Service, 800 East Beckwith Avenue, Missoula, MT 59801, USA.

[1]Corresponding author (e-mail: woodam.chung@umontana.edu).

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problem solving especially for large and complex problems (Jones et al. 1991; Weintraub et al. 1994, 1995; Martell et al. 1998; Falcão and Borges 2001; Zeki 2001; Sessions et al. 2003; Olsson and Lohmander 2005).

The prorate algorithm (Schnelle 1977) and its variations is one of the popular algorithms widely used in North America to solve the fixed- and variable-cost FTPP. Based on the shortest path algorithm (Dijkstra 1959), the prorate algorithm finds the least-cost route from each entry node (timber sale location) to the destination node (mill location) after converting the fixed cost of a link to the variable cost using the traffic volume over the link. Notable software programs using this prorate algorithm with heuristic rules include MINCOST (Schnelle 1977), NETCOST (Weintraub and Dreyfus 1985), NETWORK II (Sessions 1985), and NETWORK 2000 (Chung and Sessions 2003). Although all of these programs are able to solve large fixed- and variable-cost transportation problems, none of them can consider multiple objectives or side constraints based on other attributes of road links (i.e., limiting sediment yields, traffic volumes, or open road length).

However, modern FTPP are driven not only by the financial aspects of timber management, but also by multiple uses of roads, such as recreation, and their social and environmental impacts on such things as water quality, wildlife, and fish habitats. These environmental and social considerations and requirements introduce side constraints to the FTPP, making the problems much larger and more complex than traditional cost minimization. NETWORK 2001 (Chung and Sessions 2001) was developed to solve multiple-objective transportation-planning problems by combining a *k*-shortest path algorithm with a simulated annealing heuristic. The algorithm uses a goal-programming approach to evaluate solutions under multiple objectives, but side constraints are not considered in this problem formulation.

Except for the exact algorithms, such as MIP that has a limitation on problem size, very few algorithms have been applied to solving large fixed- and variable-cost FTPP containing side constraints. One of the promising algorithms not yet applied to FTPP is the ant colony optimization (ACO) metaheuristic, an optimization technique introduced in 1991 (Dorigo et al. 1999). The objective of our study is to apply the ACO metaheuristic concepts to develop an ACO-based heuristic algorithm to efficiently solve complex FTPP with side constraints. The ACO metaheuristic seems promising for two main reasons: (*i*) the inspiring concept of ACO metaheuristic is based on a transportation principle (it was first intended to solve transportation problems that can be modeled through networks, such as the traveling salesmen problem) and FTPP with multiple attributes of road links can be naturally modeled as a network problem, and (*ii*) ACO has been proven to be effective in finding good solutions to difficult problems as described below.

There have been successful applications of the ACO metaheuristic to solve a number of different combinatorial optimization problems. ACO algorithms are known to provide very good or the best proven results for solving many important combinatorial optimization problems such as the traveling salesman problem, quadratic assignment problem, job-shop scheduling problem, among others. Other ACO applications have provided results that matched other well-known algorithms (see Dorigo et al. 1999 and Dorigo and Stützle 2003 for more details). In forest management planning, Zeng et al. (2007) applied the ACO to a forest harvesting planning problem and found that the solution quality of the ACO was similar to that of simulated annealing and genetic algorithms.

The ACO metaheuristic has also been applied to solve network-modeled transportation problems with fixed and variable costs. For example, ACO algorithms have been developed to solve the well-known vehicle routing problem and its variants (Bell and McMullen 2004; Rizzoli et al. 2004). The most common objective function for these problems is to minimize the number of vehicle routes required to deliver products between depots and customers while minimizing the vehicle travel time or distance. Another type of network problems that have been solved using the ACO metaheuristic is the facility location problem. The objective function for these problems is set to find a subset of alternative facility locations (vertices) that minimize the combined fixed cost of facilities and the variable cost of delivering products from the selected facilities to the customer locations (Levanova 2005; McKendall and Shang 2006). Unlike the above transportation problems, the objective of FTPP is to find a subset of edges in a given network that connect timber sale locations to destinations at a minimum fixed and variable cost. Because the objective function and the constraints associated with each transportation problem are different, the direct application of the existing ACO algorithms developed for other types of transportation problems to FTPP is not viable. It is necessary to develop a specialized algorithm for FTPP, while employing the concept of the ACO metaheuristic similar to other ACO applications.

In this paper, we introduce and describe ACO-FTPP, a specially designed ACO algorithm for solving FTPP with fixed and variable costs while considering total sediment yields from the road network as a side constraint. We tested the performance of the ACO-FTPP algorithm on hypothetical 100-, 300-, and 500-edge (link) FTPP and compared the results with those obtained from solving a comparable mixed-integer mathematical programming formulation for the same problems. This mixed-integer formulation was solved using LINGO, commercially available mathematical programming software that uses the branch-and-bound algorithm (Lindo Systems Inc. 2000).

ACO metaheuristic

The ACO is a metaheuristic approach for solving difficult combinatorial optimization problems (Dorigo and Stützle 2003). ACO algorithms are inspired by the observation of the foraging behavior of real ant colonies and, in particular, by how ants can find shortest paths between food sources and the nest. When walking, ants deposit a chemical substance on the ground called pheromone, ultimately forming a pheromone trail. Although an isolated ant moves essentially at random, an ant that encounters a previously laid pheromone trail can detect it and decide with a high probability to follow it, therefore reinforcing the trail with its own pheromone. This indirect form of communication is called autocatalytic behavior, which is characterized by a positive feedback, where the more ants following a trail, the more attractive that trail becomes for being followed (Dorigo and Di Caro 1999).

The concept of the ACO metaheuristic is to set a colony of artificial ants that cooperate to find good feasible solutions to combinatorial optimization problems. Cooperation is one of the most important components of ACO algorithms. Computational resources are allocated to relatively simple agents—artificial ants. These artificial ants are an abstraction of behavioral traits of real ants, which seem to control the shortest path finding ability. In addition, artificial ants are enriched with some capabilities not present in their natural counterparts (Dorigo et al. 1999).

Four main ideas taken from real ants have been incorporated into the ACO metaheuristic (Dorigo and Di Caro 1999; Dorigo et al. 1999): (*i*) colony of cooperating ants — although each artificial ant is capable of finding a feasible solution, high-quality solutions can only emerge from the collective interaction of individuals within the entire ant colony; (*ii*) pheromone trail and indirect communication — artificial ants change some numerical information, called artificial pheromone trail, stored in the problem stage they visit, just as real ants deposit pheromone on the path they visit on the ground; (*iii*) shortest path searching and local moves — artificial ants, similar to real ones, have as a common purpose to find the shortest path moving step by step through adjacent edges; and (*iv*) stochastic and myopic state transition policy — artificial ants move through adjacent edges applying a probabilistic decision policy, which is a function of the information represented by the problem specifications (terrain conditions for real ants) and the local modifications in the problem states (by pheromone trails) induced by previous ants.

Some enriching characteristics have been given to artificial ants to increase the efficiency and efficacy of the colony. Artificial ants (*i*) live in an environment where time is discrete; (*ii*) have an internal memory of the ants' previous actions; (*iii*) deposit an amount of pheromone proportional to the quality of the solution found; and (*iv*) are not completely blind to future route options they will face and can incorporate look-ahead information, local optimization, and backtracking to improve overall system efficiency.

In ACO algorithms, a finite colony of ants concurrently and asynchronously moves through adjacent states of the problem, applying a stochastic transition policy that considers two parameters called trail intensity and visibility. Trail intensity refers to the amount of pheromone in the path, which indicates how proficient the move has been in the past, representing an a posteriori indication of the desirability of the move. Visibility is usually computed as some heuristic value indicating the a priori desirability of the move, such as cost or distance (Maniezzo et al. 2004). Therefore, moving through adjacent steps, ants incrementally build a feasible solution to the optimization problem.

Once an ant has found a solution, it evaluates the solution and deposits pheromone on the connections it used, proportionally to the goodness of the solution. Ants deposit pheromone in various ways. They can deposit pheromone on a connection (an edge in a graph) directly after the move is made without waiting for the end of the solution. This is called online step by step pheromone update. Ants also can deposit pheromone after a solution is built by retracing the same path backwards and updating the pheromone trail of the used connections. This is called online delayed pheromone update (Dorigo and Stützle 2003).

FTPP with a side constraint

The specific FTPP we address in this paper is to find the set of least-cost routes from multiple timber sales to selected destination mills while considering environmental impacts of forest road networks represented here by sediment yields. As with most transportation problems, these FTPP can be modeled as network problems containing vertices and edges. Vertices represent destination nodes (i.e., mill locations), entry nodes (i.e., timber sale locations), and intersections of road segments (links), whereas edges represent the road segments connecting these different points. Three parameters are associated with every edge: fixed cost, variable cost, and amount of sediment.

The transportation network may be composed of existing and (or) proposed roads. Fixed cost for an existing road segment is the fixed maintenance cost for the road segment but can equal zero in the absence of a relevant maintenance cost. In the case of proposed roads, the fixed cost includes the construction cost of the road segment plus a fixed maintenance cost. Fixed cost is a one-time cost that occurs if the road segment is used regardless of traffic volume. Hauling cost is a variable cost that is proportional to traffic or timber volume transported over a road segment. Although there are several ways to estimate the unit variable cost ($/volume) per road segment, it is a function of the road length, driving speed, and operating costs in most cases (Byrne et al. 1960; Moll and Copstead 1996). Because every road segment has different conditions, there will be a different unit variable cost associated with each edge. The sediment associated with each edge, expressed in tons per year per edge, represents the amount of sediment eroding from the road segment resulting from the traffic of heavy log trucks. Like fixed cost, we assume that sediment is produced when roads are open regardless of traffic volume. The Water Erosion Prediction Project (WEPP) model may be used to estimate mean annual sediment yields from each road segment in real applications (Elliot et al. 1999). Sediment yields as well as maintenance costs could also be estimated as a function of traffic volume in this problem formulation. In addition to the three parameters related to each edge, timber sale locations (origin vertices), selected mill locations (destination vertices), and timber volume per sale are required for this FTPP formulation.

The problem of finding the transportation routes that minimize the total fixed and variable costs subject to a sediment yield constraint for a single period is formulated mathematically as follows:

$$[1] \quad \text{Minimize } Z = \sum_{ab \in E} [\text{var\_cost}_{ab} \times (\text{vol}_{ab} + \text{vol}_{ba}) + \text{fixed\_cost}_{ab} \times B_{ab}]$$

subject to

$$[2] \quad \sum_{ab \in E} (\text{sed}_{ab} \times B_{ab}) \leq \text{allowable\_sed}$$

$$\text{[3.1]}\quad \text{vol\_sale}_b + \sum_{ab\in L} \text{vol}_{ab} - \sum_{ba\in L} \text{vol}_{ba} = 0 \quad \forall b \in S$$

$$\text{[3.2]}\quad \sum_{ab\in L} \text{vol}_{ab} - \sum_{ba\in L} \text{vol}_{ba} = 0 \quad \forall b \in T$$

$$\text{[3.3]}\quad \sum_{ab\in L} \text{vol}_{ab} - \sum_{b\in S} \text{vol\_sale}_b = 0 \quad b = D$$

$$\text{[4]}\quad M \times B_{ab} - (\text{vol}_{ab} + \text{vol}_{ba}) \geq 0 \quad \forall ab \in E$$

$$\text{[5]}\quad \text{vol}_{ab}, \text{vol}_{ba} \geq 0 \quad \forall ab \in E$$

$$\text{[6]}\quad B_{ab} \in \{0, 1\} \quad \forall ab \in E$$

Equation 1 specifies the objective function of the problem, where var_cost$_{ab}$ is the variable cost of wood volume transported over edge ab in either direction, vol$_{ab}$ is the wood volume transported over the edge from vertex a to b, vol$_{ba}$ is the amount transported in the opposite direction (from vertex b to a), fixed_cost$_{ab}$ is the fixed cost for edge ab in dollars, B_{ab} is a binary variable representing construction of edge ab (1 if the edge is built and 0 otherwise), and E indicates the total number of edges in the network. The first constraint (eq. 2) is the sediment constraint that limits the maximum allowable sediment amount (allowable_sed) in tons, where sed$_{ab}$ is amount of sediment from edge ab in tons if the edge is built for traffic. The second, third, and fourth sets of constraints (eqs. 3.1–3.3) ensure that all volume entering the network is channeled through the network to the destination vertex (mill location). The constraints in eq. 3.1 apply to the set of origin vertices, S (timber sale locations), and ensure the sum of sale volume entering the network at vertex b, vol_sale$_b$, plus the sum of volume transported to b from other vertices, vol$_{ab}$, equals the sum of volume transported from vertex b to connecting vertices, vol$_{ba}$. L is the set of edges having vertex b as a from-or-to node. The constraints in eq. 3.2 apply to the set of intermediate vertices, T (that are neither origin nor destination vertices), and ensure that the sum of volume entering vertex b, vol$_{ab}$, equals the sum of volume leaving that vertex, vol$_{ba}$. The constraints in eq. 3.3 apply to the destination vertex, D (mill location), and specify that the sum of the volume entering that vertex, vol$_{ab}$, equals the sum of the sale volume loaded onto the origin vertices, vol_sale$_b$, thus ensuring all volume that enters the network is routed to the destination vertex. The fifth set of constraints (eq. 4) represents the road-building constraint that makes sure that, if there is volume transported over edge ab in either direction, the edge must be constructed and open for traffic, and thus, the fixed cost and sediment amount are counted. M is a constant greater than or equal to the total amount of volume to be delivered to the mill. Lastly, the sixth and seventh sets of constraints (eqs. 5 and 6) represent the non-negativity and binary value constraints of our model, respectively.

Methodology

ACO-FTPP algorithm

ACO-FTPP is the specialized ACO algorithm we developed to solve the FTPP described above. ACO-FTPP has a finite number of ants (m) that search for s least-cost paths, one from each timber sale – destination pair, in a network of v vertices and e edges. In ACO-FTPP, a move is defined as the transition of an ant from one vertex to another. After a certain number of moves, an ant arrives at its destination, thus completing a route. Once all ants have completed their routes for one timber sale, the least-cost path is found among the m routes. Then, all ants move to the next timber sale to find m routes for that sale. An iteration is completed when all timber sales are routed to the destination vertex.

When an ant is located on a given vertex, it has to choose where to go next. An ant decides what vertex to visit next based on a random number and a transition probability on each edge calculated by the following equation:

$$\text{[7]}\quad \rho_{ab}(c) = \frac{(\tau_{ab})^{\alpha} \times (\eta_{ab})^{\beta}}{\sum_{ab\in N_l} (\tau_{ab})^{\alpha} \times (\eta_{ab})^{\beta}}$$

where $\rho_{ab}(c)$ is the transition probability with which an ant chooses edge ab in iteration c; N_l is the set of edges sharing the same origin vertex; and α and β are, respectively, parameters that control the relative importance of the pheromone trail intensity (τ_{ab}) and the visibility (η_{ab}) values on edge ab.

The visibility value (η_{ab}) is designed to evaluate the goodness of each edge in terms of the objective function and the constraint. In ACO-FTPP, η_{ab} for timber sale s is calculated by taking the reciprocal of the sum of the total variable cost, fixed cost, and sediment amount associated with edge ab (eq. 8). The total variable cost is computed by multiplying the variable cost for edge ab (\$ per unit volume) and the total volume in timber sale s:

$$\text{[8]}\quad \eta_{ab} = [(\text{var\_cost}_{ab} \times \text{vol\_sale}_s) + \text{fixed\_cost}_{ab} + \text{sed}_{ab}]^{-1}$$

Consequently, by combining eqs. 7 and 8, the resulting transition probability formula for a given edge is

$$\text{[9]}\quad \rho_{ab}(c) = \frac{(\tau_{ab})^{\alpha} \times \{[(\text{var\_cost}_{ab} \times \text{vol\_sale}_s) + \text{fixed\_cost}_{ab} + \text{sed}_{ab}]^{-1}\}^{\beta}}{\sum_{ab\in N_l} (\tau_{ab})^{\alpha} \times \{[(\text{var\_cost}_{ab} \times \text{vol\_sale}_s) + \text{fixed\_cost}_{ab} + \text{sed}_{ab}]^{-1}\}^{\beta}}$$

Each ant generates a random number for each connecting edge in N_l. Then, an edge from N_l is selected as the next move of the ant based on the transition probability and the selected random number. The higher transition probability an edge has, the better is the chance of it being selected.

Starting from a given s and ending at the mill destination, an ant incrementally builds a route, moving through adjacent edges according to the transition probability (eq. 9). At the end, the best route among the m routes generated by the ants is selected as the least-cost path. At the end of each iteration, the edges forming all least-cost paths (one for every sale–destination pair) are identified, the objective function value is computed, and the solution feasibility is evaluated. If the current solution is not better than the best found so far or is infeasible, the solution is ignored, the pheromone trail intensities remain the same, and another iteration starts. However, if the current solution is better than the best solution found so far, the current solution becomes the new best solution, and the pheromone trail intensity of the edges forming all least-cost paths is updated. At the same time, pheromone intensity on all edges decreases (evaporates) to avoid unlimited accumulation of pheromone.

Pheromone evaporation avoids a too-rapid convergence of the algorithm towards a suboptimal solution, allowing the exploration of other areas of the solution space. Pheromone trail intensity is updated using the following equation:

[10] $$\tau_{ab}(c+1) = \lambda \times \tau_{ab}(c) + \Delta\tau_{ab}$$

where two components are considered. Firstly, the current pheromone trail intensity on edge ab at iteration c, represented by $\tau_{ab}(c)$, is multiplied by $0 < \lambda < 1$, where $1 - \lambda$ represents the pheromone evaporation rate between iteration c and $c + 1$. Secondly, $\Delta\tau_{ab}$ represents the newly added pheromone amount to edge ab and is calculated as

[11] $$\Delta\tau_{ab} = \sum_{k=1}^{S} \Delta\tau_{ab}^{k}$$

where $\Delta\tau_{ab}^{k}$ is the quantity of pheromone laid on edge ab by the ants in iteration c, which is given by

[12] $$\Delta\tau_{ab}^{k} = \begin{cases} Q/L_k & \text{if the ants used edge } ab \text{ in the } k\text{th least cost path} \\ 0 & \text{otherwise} \end{cases}$$

where Q is a constant and L_k is the total transportation cost over the kth least-cost path. The value of Q has to be chosen so the amount of pheromone added to edge ab by a given ant slightly increases the probability of selection of that edge in the next iteration.

Given the definitions above, the ACO-FTPP solution process can be described as follows (Fig. 1). During iteration 1, an initialization phase takes place in which ants representing the timber sale locations start in random order. An initial equal small amount of pheromone q is set for each edge, and transition probabilities for each edge are computed considering the volume of the chosen timber sale. Thereafter, each ant can find a route by moving from edge to edge until the mill destination is reached.

When an ant moves through an edge, the edge is recorded along with its from and to vertex in the ant's internal memory. This memory is used to avoid ants returning to a previously visited vertex. When an ant arrives at a vertex whose adjacent vertices have all been previously visited, it stops without reaching its destination and a high cost (i.e., \$ 999 999) is assigned to the ant's route as a penalty. Likewise, if an ant has not found its destination after a maximum number of moves, Max_moves, the ant stops, and a high cost is assigned. For the applications used in this paper, Max_moves is set to be the number of vertices in the network plus one (v + 1).

After every ant finds its own route, the least-cost path is selected, and all ants move to the next randomly chosen sale (origin). Ants start moving through adjacent edges until they find the destination mill. When the least-cost path is selected for this second sale, all ants move to the next sale, and so forth. The iteration ends when routes from all sales have been analyzed, then the objective function and total sediment values are calculated using the least-cost path for each timber sale. The edges forming the s least-cost paths (one per timber sale) are identified, and their pheromone trail intensity is updated for the next iteration when a better solution is found. This iterative process continues until a stopping criterion is met. We used a maximum number of iterations, $I_{\max}$, to stop the process in a reasonable time.

Hypothetical transportation problems

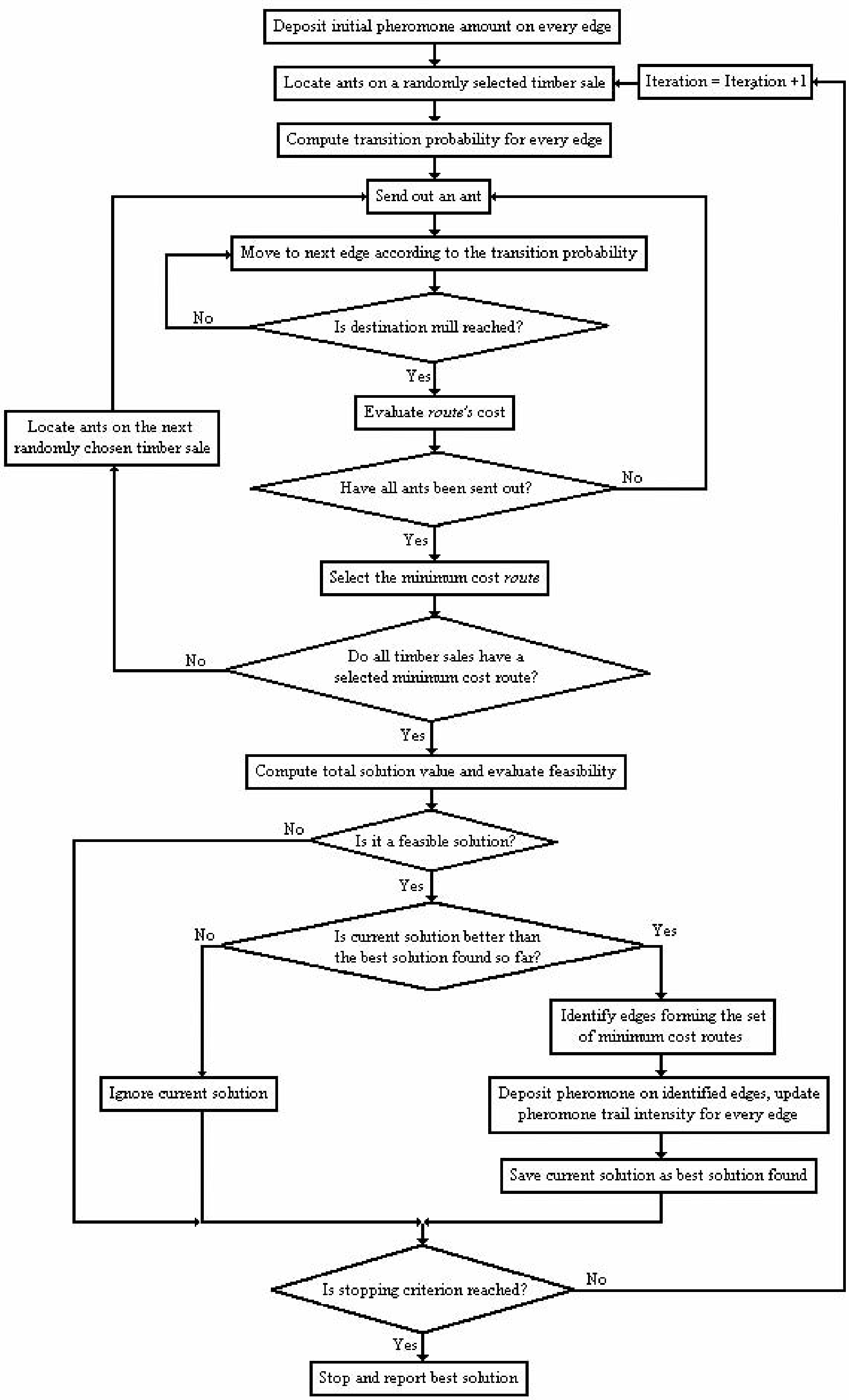
To examine the behavior and performance of the algorithm, we tested the ACO-FTPP using three hypothetical forest transportation problems that include 100, 300, and 500 edges, respectively (Figs. 2–4). Variable cost, fixed cost, and sediment amount per edge in the three problems range from \$0.01/m^3 to \$10/m^3 of wood hauled, from \$0.1 to \$ 23 000 for road construction and maintenance, and from 0.4 to 200 t of sediment, respectively. The 100-edge FTPP includes five timber sales with a total volume of 3850 m^3 of wood to deliver. Volume per timber sale ranges between 670 and 860 m^3. The 300-edge and 500-edge problems include 15 and 25 timber sales with total volumes of 16 700 and 36 500 m^3, respectively. The highest and lowest volumes given to a single timber sale are 750 and 1700 m^3, respectively, for the 300-edge problem, and 1030 and 1900 m^3 for the 500-edge problem.[2]

According to our model formulation, problem complexity can be described in terms of the number of variables and constraints. In general, our formulation includes ($3 \times e$) variables and ($e + v + 1$) constraints, where e and v are the total number of edges and vertices in the road network. Therefore, the three hypothetical problems have 300, 900, and 1500 variables and 141, 421, and 701 constraints, respectively.

The three hypothetical problems were designed to be challenging to solve to provide a rigorous test of ACO-FTPP. These hypothetical problems form grid-shaped road networks that are known to be more difficult to solve than tree-shaped road networks that exist in most real-world forest road systems (Andalaft et al. 2003). These hypothetical, grid-shaped problems contain many circuits resulting in many possible paths from a given entry node to a destina-

[2] The input data for the three hypothetical FTPP can be downloaded at ftp://ftp.forestry.umt.edu/special/chung/download/AntColony/.

Fig. 1. Flowchart of the ACO-FTPP search process.



tion, and solution difficulty is a function of the number of available options. On the contrary, tree-shaped road networks have obvious loaded-truck directions and numerous dead-end road sections, making fewer edge–direction combinations to analyze. Also, real forest road networks do not often have intersections where four or even more road segments meet. An increasing number of road segments leaving an intersection point dramatically increases the number of possible paths. The degree of a vertex is defined as the number of adjacent edges. In our hypothetical examples, the

Fig. 2. Hypothetical forest transportation problem with 100 edges and 5 timber sales.

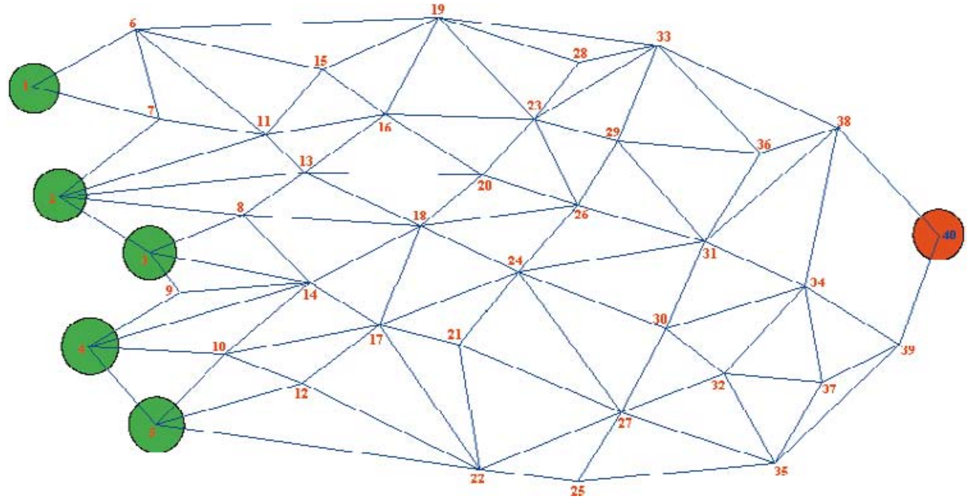
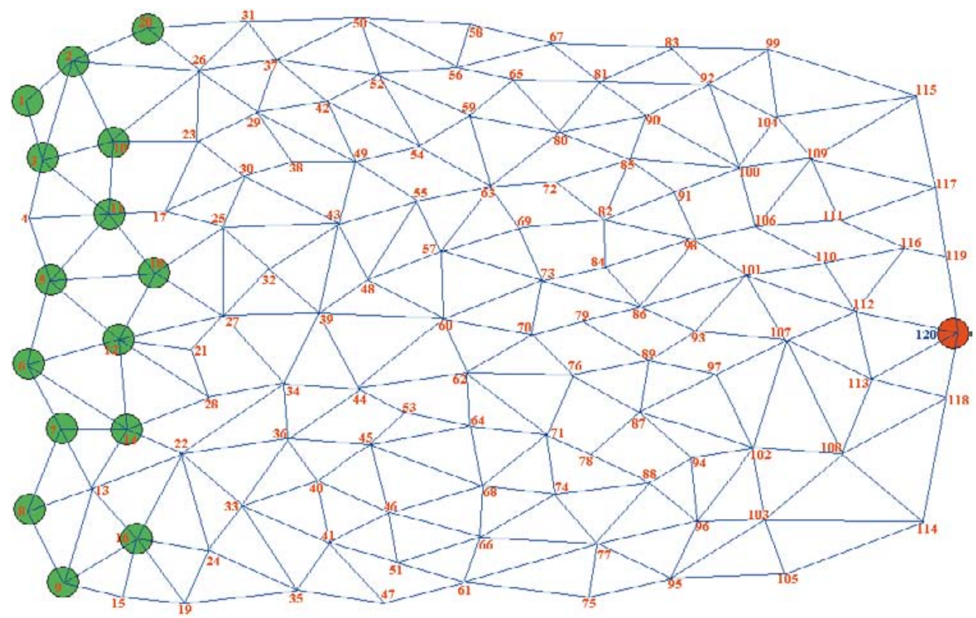


Fig. 3. Hypothetical forest transportation problem with 300 edges and 15 timber sales.



minimum degree is two (i.e., vertices 1 and 40 in Fig. 2), the maximum is seven (i.e., vertices 14 and 17 in Fig. 2), and the mean degree of the graphs representing these transportation problems is five, whereas it is rare to find road junctions where the degree of a vertex is larger than three in real-world forest road networks.

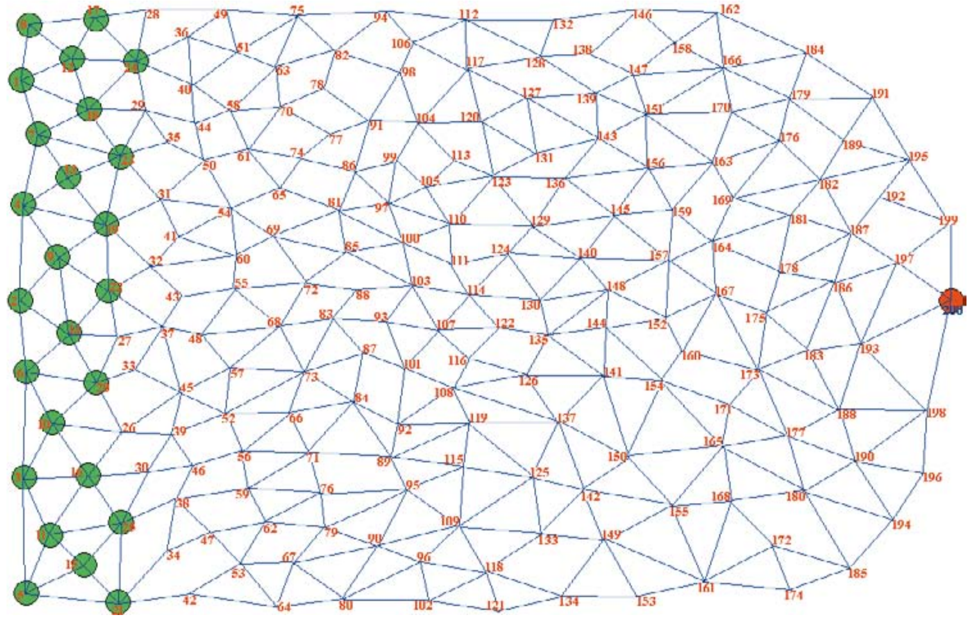
Setting parameters

ACO-FTPP requires values for the parameters α, β, λ, q, Q, m, and $I_{\max}$. Our initial test runs of ACO-FTPP confirmed the findings of previous studies that different parameter combinations affect the performance of the ACO (Dorigo et al. 1996). We conducted a search for the best ACO-FTPP parameter combination using the 100-edge problem. Because several parameter combinations among the many we tested could find the same best solution, we used algorithm efficiency as well as solution quality as the criteria for selecting the best parameter combination. Efficiency was measured in terms of the number of iterations taken to find the best solution.

Three of the seven parameters required by ACO-FTPP (q, m, and $I_{\max}$) do not affect the calculation of the transition probability (eqs. 7–12). Therefore, these parameter values were fixed in our trials. The parameter q does not affect the ants search because it represents a small amount of pheromone deposited at time zero, $\tau_j(0)$, on every edge (Dorigo et al. 1996). In most ACO algorithms, q is set to a small positive constant. For our applications, q was set to 0.001. Similarly, m is usually set to v (Dorigo et al. 1996). Because our FTPP are complex problems that consider three variables associated with every edge instead of one m was set equal to e, which is larger than the number of vertices, to diversify the search in our applications. Based on initial runs, $I_{\max}$ was set to give the algorithm enough time to find the best solution; in our applications, $I_{\max}$ was set to 100.

Parameters Q, α, β, and λ directly affect the calculation of

Fig. 4. Hypothetical forest transportation problem with 500 edges and 25 timber sales.



the transition probability (eqs. 7–12); therefore, they may have an important effect on algorithm performance. The constant Q, which is related to the quantity of pheromone deposited by ants, has to be chosen so the transition probability of an edge is slightly increased over iterations. Because our initial test runs showed that Q did not have a significant effect on the solution quality, we set Q to 0.001. The remaining parameters (α, β, and λ) were identified to directly affect the performance of the algorithm and, therefore, subject to the search for the best parameter combination.

To test different values of the parameters α, β, and λ, a range for each parameter was defined and partitioned into 10, 15, and 10 discrete values, respectively. Table 1 shows the range of values and the corresponding discrete values tested for each parameter. This yields 1500 different parameter combinations. We solved the 100-edge problem (Fig. 2) using each of these combinations. It took about 50 min to conduct 1500 runs and find the best parameter combination.

The best parameter combination found by this search was $\alpha = 1.5$, $\beta = 0.16$, and $\lambda = 0.75$. We noticed from our runs that η values were relatively very large compared with the τ values. The best parameter combination selected shows that $\beta < 1.0$ was chosen to lower the contribution of visibility value to the transition probability. Consequently, the relative importance of η and τ are better balanced. The best value found for λ (0.75) may be explained by the fact that the ants need to "forget" part of the experience gained in the past, represented by the accumulated pheromone amount, to better exploit new incoming pheromone information and to avoid a fast convergence to suboptimal solutions. Dorigo et al. (1996) observed the same behavior in their parameter setting procedure.

Our test runs on the 100-edge problem indicated that β has a larger effect on solution quality than α or λ (see Fig. 8). Thus, to find good parameter combinations for the 300- and 500-edge problems and reduce computation time required for the search, we ran the ACO-FTPP with various

Table 1. Range of values for the parameters that control the relative importance of the pheromone trail intensity (α) and visibility (β), and the evaporation rate (λ).

Parameter	Value range	Discrete values
α	$0 < \alpha < 10$	0.5, 1.5, 2.5, …, 9.5
β	$0 < \beta < 0.3$	0.02, 0.04, 0.06, …, 0.30
λ	$0 < \lambda < 1$	0.05, 0.15, 0.25, …, 0.95

values of β (Table 1) while holding α and λ to the values used for the 100-edge problem, $\alpha = 1.5$ and $\lambda = 0.75$. From these trials, the best values were $\beta = 0.22$ for the 300-edge problem and $\beta = 0.28$ for the 500-edge problem. It took approximately 4.5 and 40.5 min to conduct the searches for the 300- and 500-edge problems, respectively. These parameters were then used for all four problem cases analyzed in each problem size category.

Test cases

Four cases were analyzed for each of the three hypothetical examples to test the ACO-FTPP algorithm. Case I was a cost minimization problem without a sediment constraint, cases II and III were cost-minimization problems subject to increasing levels of upper-bound sediment constraints, and case IV was a sediment-minimization problem without a cost constraint. Whereas cases I and IV address single-goal transportation-planning problems, cases II and III address multiple goals represented by one objective and one side constraint.

Once case I was solved, the minimum cost solution was obtained, and the associated total sediment amount was calculated. This sediment amount became the upper limit for establishing the sediment constraints for cases II and III because any larger sediment constraint values would not affect the minimum cost solution. Case IV provided the lower limit for the sediment constraint because requiring sediment below that limit would result in an infeasible solution. The sediment constraint values for cases II and III were set be-

Table 2. Comparisons on the objective function values between the ACO-FTPP and MIP solutions for the 100-edge, 300-edge, and 500-edge hypothetical FTPP.

Case	100 edges			300 edges			500 edges		
	ACO-FTPP objective function	MIP objective function	Difference between ACO-FTPP and MIP (%)	ACO-FTPP objective function	MIP objective function	Difference between ACO-FTPP and MIP (%)	ACO-FTPP objective function	MIP objective function	Difference between ACO-FTPP and MIP (%)
I	128 057	128 057	0.00	711 309	702 558	1.25	1 530 227	1 496 979	2.22
II	151 290	151 290	0.00	732 194	712 309	2.79	1 667 287	1 585 881	5.13
III	185 701	178 921	3.79	811 143	787 499	3.00	2 055 796	2 054 435[b]	0.07
IV	393.67	393.67	0.00	605.97	571.84	5.97	970.30	948.58[b]	2.29

[a]The units of the objective function value for cases I through III are dollars and for case IV are tons of sediment.
[b]Objective function value of the best feasible solution found by the MIP solver after 336 h of computing time.

tween the upper and lower limits obtained by cases I and IV. The level of the sediment restriction is increased from case II to case III.

To efficiently guide ants in their search for the least-cost path, the transition probability function (eq. 9), which contains fixed and variable costs and sediment amounts, was modified for the objectives in unconstrained cases I and IV. For case I, the transition probability considered only the variable and fixed costs associated with each edge using the following equation:

$$[13] \qquad \rho_{ab}(c) = \frac{(\tau_{ab})^{\alpha} \times \{[(\text{var\_cost}_{ab} \times \text{vol\_sale}_{s}) + \text{fixed\_cost}_{\text{ab}}]^{-1}\}^{\beta}}{\sum\limits_{ab \in N_l} (\tau_{ab})^{\alpha} \times \{[(\text{var\_cost}_{ab} \times \text{vol\_sale}_{s}) + \text{fixed\_cost}_{\text{ab}}]^{-1}\}^{\beta}}$$

For case IV, the transition probability considered only the sediment amount associated with each edge using the following equation:

$$[14] \qquad \rho_{ab}(c) = \frac{(\tau_{ab})^{\alpha} \times (\text{sed}_{ab}^{-1})^{\beta}}{\sum\limits_{ab \in N_l} (\tau_{ab})^{\alpha} \times (\text{sed}_{ab}^{-1})^{\beta}}$$

Model verification

The ACO, like any heuristic, does not guarantee optimal solutions. To verify the results of our algorithm and to obtain a measure of solution quality, we compared ACO-FTPP solutions with results obtained by solving the mixed-integer mathematical programming (MIP) formulation described by eqs. 1–4, which is known to be well suited for this type of transportation problems (Kim and Hooker 2002). The MIP model was formulated and solved using LINGO version 10.0, which employs the branch-and-bound algorithm (Lindo Systems Inc. 2000). The default LINGO optimization setting parameters were used.

Results and discussion

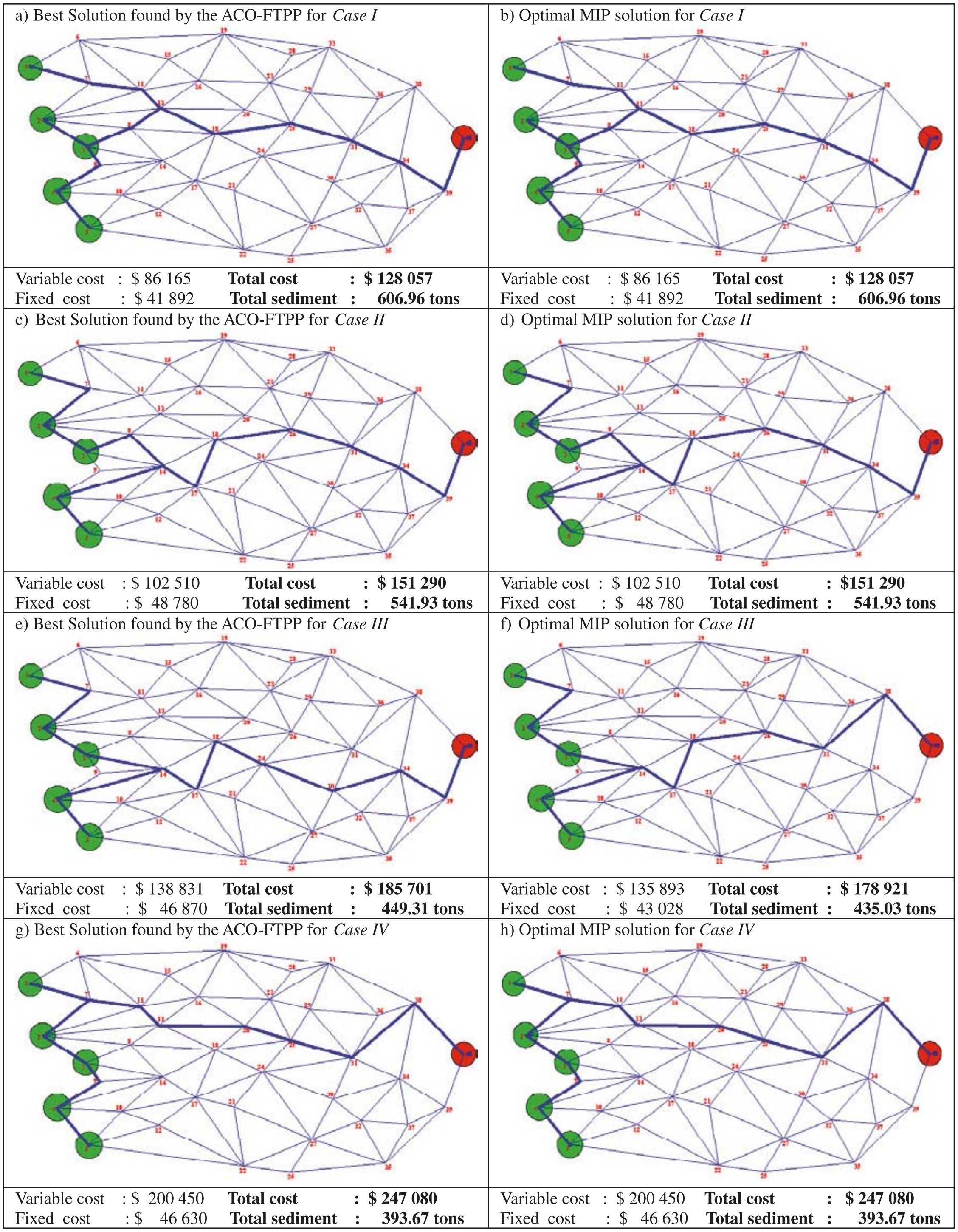
The ACO-FTPP was able to find feasible solutions for all four cases for each of the three hypothetical problems. The MIP solver was run to optimality for all cases in the three hypothetical problems, except for cases III and IV of the 500-edge problem. Although we ran the MIP solver for up to 2 weeks (336 h) for each of those two cases, the solver could not reach the optimal solution. However, it did report the best feasible solution found during that computation time period.

The resulting objective function values and constraint values are compared between the ACO-FTPP and MIP approaches in Table 2. For the 100-edge hypothetical FTPP, best solutions found by the two problem-solving approaches are compared in Figs. 5*a* through 5*h*. For case I and case II, both problem-solving approaches resulted in the same solution. The solution found for case I, the unconstrained cost minimization problem, has an objective function value of \$128 057 and the total associated sediment amount of 606.96 tons (Figs. 5*a* and 5*b*). The objective function value and the sediment amount for case II, where the maximum allowable sediment value was set to 550 tons, are \$151 290 and 541.93 tons, respectively (Figs. 5*c* and 5*d*). For case III, where the sediment restriction was set to 450 tons, the best ACO-FTPP solution has an objective function value of \$185 701, which is \$6780 higher than the MIP solution, while still meeting the sediment constraint (Figs. 5*e* and 5*f*). For case IV, where only sediment yields were minimized, the best solution found by ACO-FTPP again matched the MIP solution with the total sediment amount of 393.67 tons (Figs. 5*g* and 5*h*).

The results from the 300-edge hypothetical FTPP are presented in Fig. 6. All four cases analyzed show the ACO-FTPP solutions have higher total costs than the MIP solutions. For case I, the objective function value of the ACO-FTPP solution was \$8751 higher than the MIP solution (Figs. 6*a* and 6*b*). For cases II and III where sediment constraints were set at 1000 and 800 t, respectively, the ACO-FTPP solutions were \$ 19 885 and \$ 23 644 higher, respectively (Figs. 6*c*–6*f*). For case IV, the objective function value of the ACO-FTPP solution was 34.13 tons higher than the MIP solution (Figs. 6*g* and 6*h*).

Figures 7*a*–7*h* compare the results from the two problem-solving approaches for the 500-edge hypothetical FTPP.

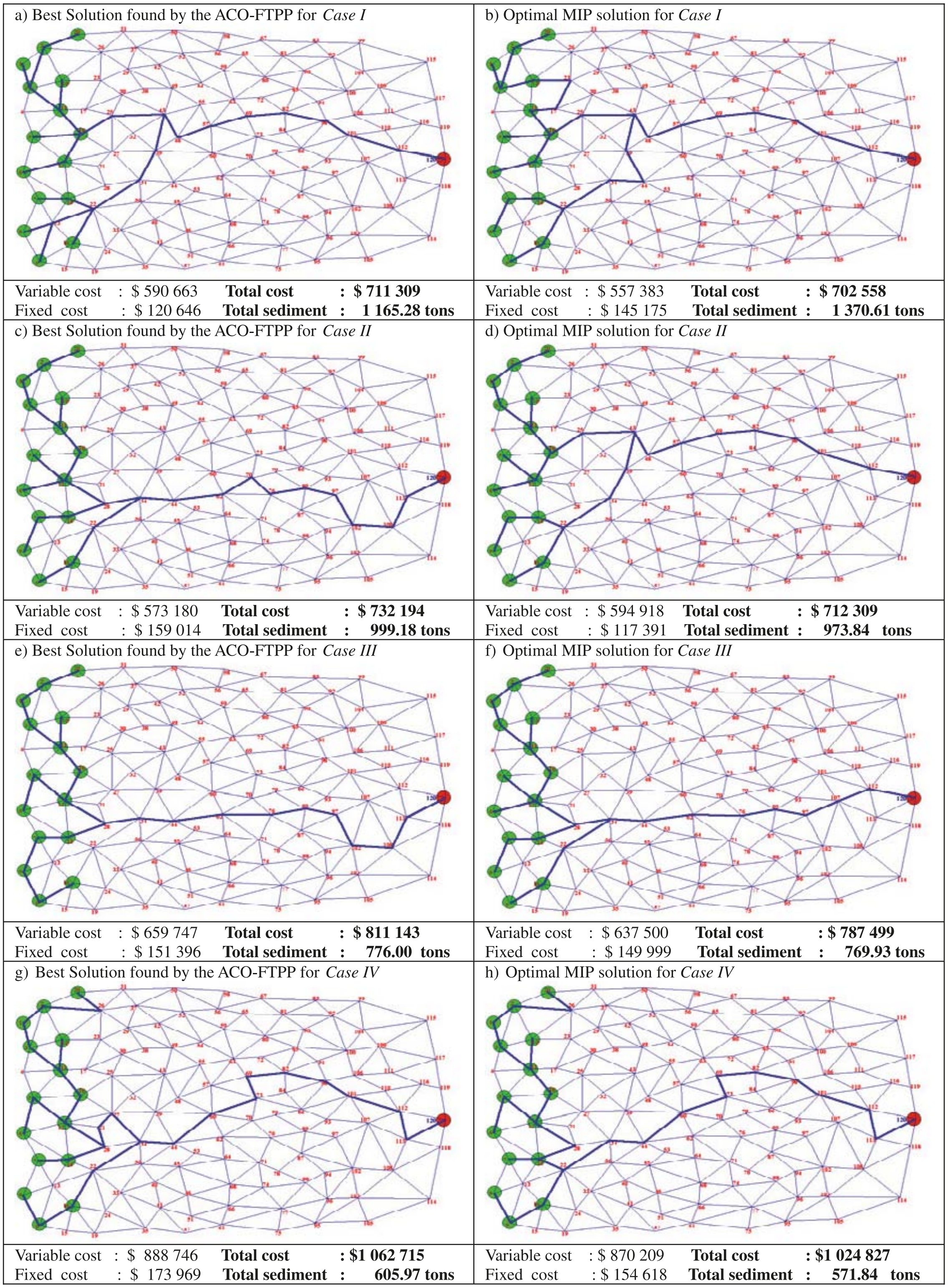
Fig. 5. Solution comparisons between ACO-FTPP and the MIP solver for the 100-edge FTPP.



Like the 300-edge problem, all four cases analyzed show that solutions found by the MIP solver (either optimal or feasible solutions) are slightly better than the ACO-FTPP solutions. The maximum allowable sediment amounts set for case II and III were 2000 and 1500 tons, respectively. The magnitude and percentage differences in objective function values between the two approaches were $ 33 248 (2.2%), and $ 81 406 (5.1%), respectively, for cases I and II. For cases III and IV, where the MIP solver could not find the optimal solution, ACO-FTPP solutions were slightly higher by $1361, and 21.72 tons, respectively, than the best solution found by the MIP solver.

Compared with the optimal solutions, the optimality level of the ACO-FTPP solutions was at least 94% (Table 2). There is no certainty about the optimality level of ACO-FTPP solutions for cases III and IV of the 500-edge problem because the MIP solver was not able to find an optimal solution in a reasonable amount of time. However, the ACO-

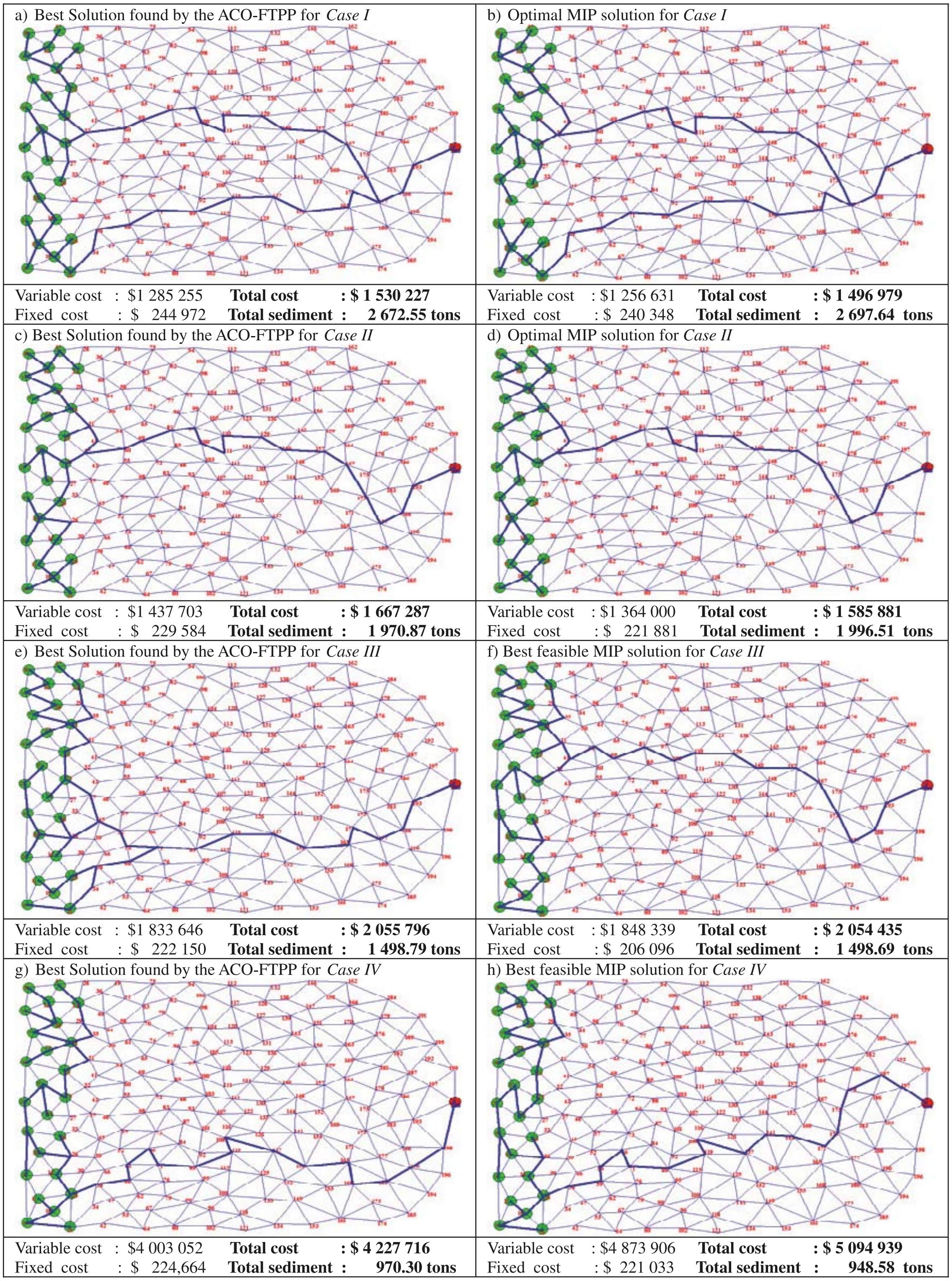
Fig. 6. Solution comparisons between ACO-FTPP and the MIP solver for the 300-edge FTPP.



FTPP solutions are within 2.3% of the best feasible MIP solutions found. A general trend may be seen in the results that the solution quality of ACO-FTPP may decrease as problem size increases (i.e., 100 edges vs. 500 edges) or the constraint becomes more binding (i.e., case II vs. case III).

The ACO-FTPP algorithm was implemented in the C programming language and run on a 3.40 GHz Pentium(R) D computer with 2.00 GB of RAM. For comparisons on computation time, the same computer was used to run the MIP solver. The computation times taken by ACO-FTPP and the MIP solver are compared in Table 3. The ACO-FTPP computation time was constant across the cases within each indi-

Fig. 7. Solution comparisons between ACO-FTPP and the MIP solver for the 500-edge FTPP.



vidual problem because the number of ants and iterations were constant for all cases within each hypothetical problem. However, the computation time required for ACO-FTPP increased with problem size because the number of ants increased with the total number of edges. In contrast, the computation time required for the MIP solver varied among different cases and increased as the problem size became larger.

In most cases, ACO-FTPP solved problems much faster than the MIP solver. For example, for case II in the 500-edge problem, ACO-FTPP took only a fraction of the computation time that was required for the MIP solver (2 min,

Table 3. Comparisons on the computation time (h:min:s) for a single run required by ACO-FTPP and the MIP solver for the 100-edge, 300-edge, and 500-edge hypothetical FTPP.

	100 edges		300 edges		500 edges	
Case	ACO-FTPP	MIP solver	ACO-FTPP	MIP solver	ACO-FTPP	MIP solver
I	00:00:02	00:00:01	00:00:18	00:02:25	00:02:42	00:05:35
II	00:00:02	00:00:05	00:00:18	00:06:36	00:02:42	54:38:53
III	00:00:02	00:00:09	00:00:18	01:35:48	00:02:42	336:00:00
IV	00:00:02	00:00:10	00:00:18	00:24:19	00:02:42	336:00:00

42 s vs. over 54 h). Moreover, the MIP solver was not able to find an optimal solution after 336 h of computation time for cases III and IV, whereas ACO-FTPP took only 0.045 h to find feasible and relatively good solutions which are within 0.07% and 2.29% from the MIP best solutions for cases III and IV, respectively. It seems that both approaches take more computation time as problem size increases, but computation time for the MIP solver increases much more dramatically than ACO-FTPP.

Algorithm sensitivity to parameter values

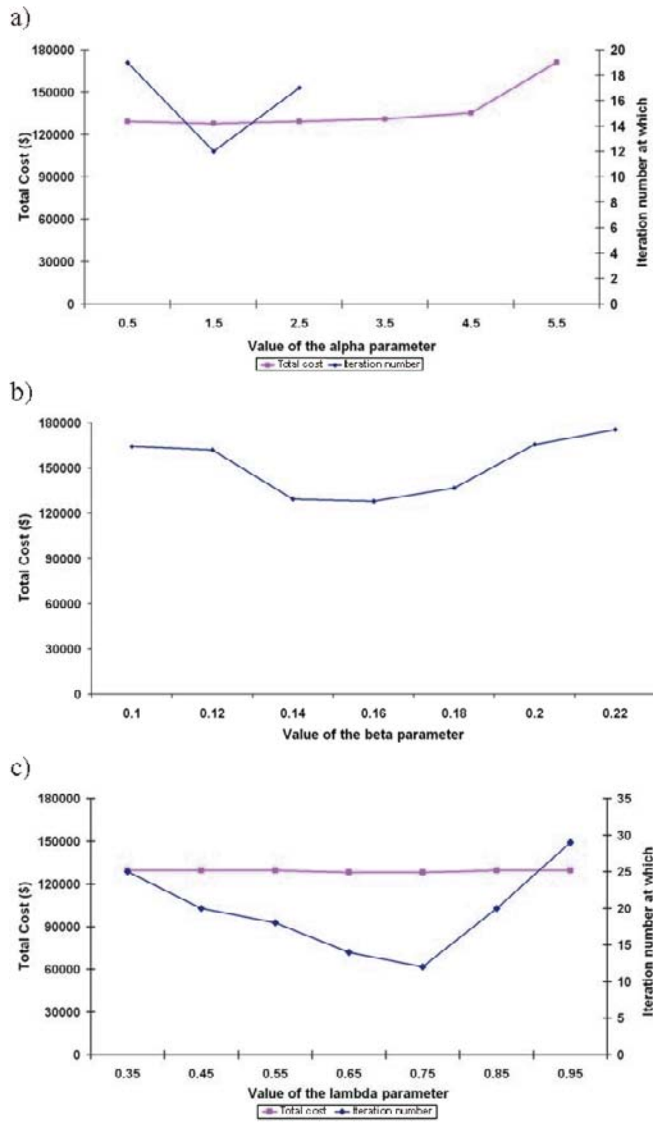
To evaluate the effects of small parameter changes on the algorithm performance, sensitivity analyses were carried out for α, β, and λ using case I of the 100-edge problem. A range of values for each of α, β, and λ were tested while the other parameters were held constant. The default constant values used for α, β, and λ were 1.5, 0.16, and 0.75, respectively (the best parameter combination found previously).

The tested values for α were 0.5, 1.5, 2.5, 3.5, 4.5, and 5.5. Figure 8*a* shows how the solution quality changes across the different values of α. When α was in the range of 0.5 to 2.5, the objective function values were similar, although a slightly better solution was found when α was 1.5. However, the number of iterations required to find the best solution increased when α deviated from 1.5. When α became >4.5, the ACO-FTPP algorithm failed to find good-quality solutions; this indicates that, when the pheromone trail intensity dominates the transition probability, the ACO algorithm may get easily trapped in a local optimum.

The solution quality is also affected by β (Fig. 8*b*). The tested values for β ranged from 0.10 to 0.22 in increments of 0.02. The results show that, as β deviated from 0.16, the solution quality became increasingly degraded because, again, the balance between the pheromone intensity and visibility was no longer achieved.

Lastly, we tested several values $1 - \lambda$ from 0.35 to 0.95 in increments of 0.1 (Fig. 8*c*). The same best solution was found when λ was 0.65 and 0.75, whereas a slightly worse solution was found with the other values of λ. However, the number of iterations taken to reach the best solution continuously changed across the tested λ values, which indicates that the pheromone evaporation rate is another important factor that may largely affect the algorithm efficiency as well as the solution quality.

Fig. 8. Algorithm sensitivity to the parameter (*a*) alpha (α) that controls the relative importance of pheromone trail intensity, (*b*) beta (β) that controls the relative importance of the visibility, and (*c*) lambda (λ) that controls the evaporation rate.



Conclusions

In this paper, we introduced the use of ACO metaheuristic approach as an optimization technique in forest transportation planning and developed an ACO-based heuristic algorithm (ACO-FTPP) to solve large forest transportation problems with side constraints. The ability to consider side constraints in transportation planning problems enables analysts to address various transportation and environmental issues (e.g., mill capacity, traffic control, and environmental impacts of roads) that are otherwise difficult to consider in the planning process.

The tests on complex, grid-shaped networks reported in this paper indicated that the ACO-FTPP produces near-optimal, minimum-cost solutions for transportation planning problems containing both fixed and variable costs as well as a binding side constraint. The difference between the objective function values produced by ACO-FTPP and MIP did increase as problem size increased but were still within 6% of the MIP solution for the largest network problems tested. Moreover, the solution times for ACO-FTPP increased from seconds for the small network problems to only minutes for the largest network problems tested. In comparison, the MIP solution times increased from seconds for the small network to many hours for the large network, and MIP was unable to find the optimal solution in two of the four large-network cases. The observed modest changes in solution times as problem size increased as well as high solution quality suggest that the ACO-FTPP has good potential as a generalized algorithm for efficiently solving large, complex, real-world FTPP.

The current problem formulation of the ACO-FTPP is based on a single time period. Future study should address multiple time periods while paying careful attention to solution quality, because handling multiple periods increases problem size substantially and adds complexity to the problem. To address multiple periods, the current problem formulation can be conceptually modified as follows: (*i*) all of the future fixed and variable costs should be discounted to present values; (*ii*) multiple sets of decision variables associated with each edge need to be developed to represent multiple time periods; (*iii*) conservation of flow, road building, and sediment amount constraints for each time period should be added; and (*iv*) new constraints are necessary to restrict the use of each edge before it is constructed.

The hypothetical problems used in this study have a wide range of road attribute values (e.g., costs and sediment yields). Although exploring the algorithm performance as a function of the range of road attribute values was outside the scope of our study, one might be interested in understanding the algorithm sensitivity to different ranges of attribute values. We anticipate that the ACO-FTPP algorithm would also work well for problems with a narrow range of attribute values when proper algorithm parameters are selected and used. The effects of differences in attribute values on transition probability can be easily enhanced or diminished by the algorithm parameters, particularly α and β in eq. 9. Like other heuristic solution approaches, customizing algorithm parameters to specific problems will be crucial for successful applications of the ACO-FTPP algorithm.

Further development of the algorithm is suggested in the following four areas to enhance its performance as a generalized approach for solving large FTPP containing side constraints. Firstly, the road attributes used to evaluate the transition probability associated with each edge (fixed cost, variable cost, and sediment amount) could be standardized to a mean of zero and a variance of one. This would avoid the magnitude of attribute values (as affected by unit of measure differences, for example) affecting evaluation of the transition probability equations that are used to predict the goodness of a road segment in the solution. Secondly, local search techniques such as the 2-opt heuristic can also be combined with ACO-FTPP to improve solution quality, although it may likely increase the computation time. The 2-opt heuristic is an exhaustive search of all permutations obtainable by exchanging two edges adjacent in solution found at the end of each iteration. Thirdly, to increase the number of feasible solutions obtained by ACO-FTPP, the variables representing the side constraints could have a more active role in the solution building process. That is to say, besides only affecting the transition probability; they could be designed to predict solution feasibility through look-ahead functions. Fourthly, because the optimal algorithm parameters vary depending on the nature and size of the problem, further evaluation on the robustness of the parameters should be conducted by applying ACO-FTPP to various problem types. As shown in the sensitivity analyses, the proper tuning of parameters can significantly improve the solution quality as well as the computational efficiency of the algorithm.

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